

Three main types of trigonometric identities. Reciprocal, Quotient, Pythagorean

### Reciprocal Identities

$\sin \theta = \frac{1}{\csc \theta}$	$\csc \theta = \frac{1}{\sin \theta}$
$\cos \theta = \frac{1}{\sec \theta}$	$\sec \theta = \frac{1}{\cos \theta}$
$\tan \theta = \frac{1}{\cot \theta}$	$\cot \theta = \frac{1}{\tan \theta}$

### Quotient Identities

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

Rewrite with opp, adj, and hyp

$$\frac{\cancel{\text{opp}}}{\cancel{\text{hyp}}} \cdot \frac{\cancel{\text{hyp}}}{\cancel{\text{adj}}}$$

Simplify fraction

$$\frac{\text{opp}}{\text{hyp}} \cdot \frac{\text{hyp}}{\text{adj}}$$

Rewrite as one trig function

$$\frac{\text{opp}}{\text{adj}} = \tan \theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Type III: Pythagorean Identities

Pythagorean Theorem:

$$\frac{a^2 + b^2 = c^2}{x^2 + y^2 = r^2}$$

$$\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$$

Rewrite using x,y,r:

$$\frac{\cos^2 \theta + \sin^2 \theta = 1}{\cos^2 \theta + \sin^2 \theta = 1}$$

\*\*1<sup>st</sup> identity

Rewrite using trig:

$$\frac{\cot^2 \theta + 1 = \csc^2 \theta}{\cot^2 \theta + 1 = \csc^2 \theta}$$

\*\*2<sup>nd</sup> identity

Divide by  $\sin^2 \theta$ :

$$\frac{1 + \tan^2 \theta = \sec^2 \theta}{1 + \tan^2 \theta = \sec^2 \theta}$$

\*\*3<sup>rd</sup> identity

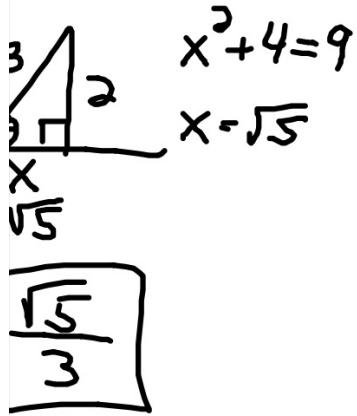
Divide 1<sup>st</sup> identity by  $\cos^2 \theta$ :

$$r^2 = \frac{1}{\sin^2 \theta}$$

$\cos^2 \theta + \sin^2 \theta = 1$	$\cot^2 \theta + 1 = \csc^2 \theta$	$1 + \tan^2 \theta = \sec^2 \theta$
$\cos^2 \theta = 1 - \sin^2 \theta$	$\cot^2 \theta = \csc^2 \theta - 1$	$\tan^2 \theta = \sec^2 \theta - 1$
$\sin^2 \theta = 1 - \cos^2 \theta$	$1 = \csc^2 \theta - \cot^2 \theta$	$1 = \sec^2 \theta - \tan^2 \theta$

e for the values  
in the 1<sup>st</sup> quadrant.

$$\sin \theta = \frac{2}{3}, \text{ find } \cos \theta.$$



Express each as a function  
of a 1<sup>st</sup> quadrant angle.

$$2. \cos 892^\circ =$$

$$\cos 172^\circ$$

$$-\cos 8^\circ$$

Simplify to one trig fun

$$3. \frac{\sin^2 \theta \cot \theta}{\cos \theta} =$$

$$\frac{\sin \theta \cdot \frac{\cos \theta}{\sin \theta}}{\cos \theta}$$

$$\frac{\sin \theta \cos \theta}{\cos \theta}$$

$$\sin \theta$$

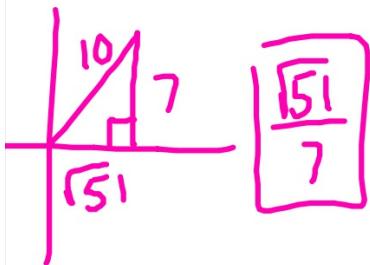
**e Problems:**

or the values in the 1<sup>st</sup> quadrant.

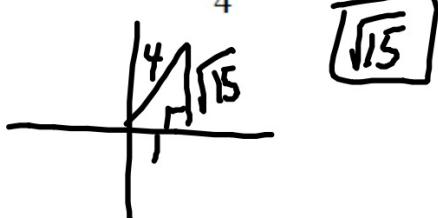
$\theta = 2$ , find  $\cot \theta$ .

$$\frac{1}{2}$$

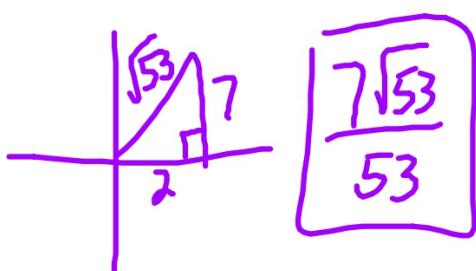
$\theta = \frac{7}{10}$ , find  $\cot \theta$ .



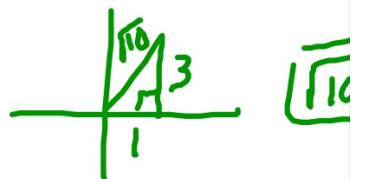
5. If  $\cos \theta = \frac{1}{4}$ , find  $\tan \theta$ .



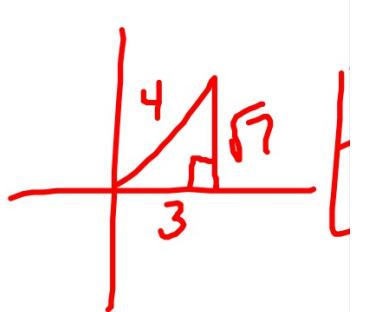
8. If  $\tan \theta = \frac{7}{2}$ , find  $\sin \theta$ .



6. If  $\tan \theta = 3$ , find  $\sec \theta$ .



9. If  $\sec \theta = \frac{4}{3}$ , find  $\sin \theta$ .



is each function as a function of a 1<sup>st</sup> quadrant angle.

$$1458^\circ =$$

$$\sin 98^\circ$$

$$\sin 82^\circ$$

$$11. \tan(-876^\circ) =$$

$$\tan 204^\circ$$

$$\tan 24^\circ$$

$$12. \csc 495^\circ =$$

$$\csc 135^\circ$$

$$\csc 45^\circ$$

fy to one trig function.

$$\frac{\sin \theta}{\cos \theta} =$$

$$\frac{\cos \theta}{\sin \theta}$$

$$\frac{\sin \theta}{\cos \theta}$$

$$\frac{\cos \theta}{\cos \theta}$$

$$\cot^2 \theta$$

$$14. \sin^2 \theta \cos^2 \theta - \cos^2 \theta$$

$$\cos^2 \theta (\sin^2 \theta - 1)$$

$$\cos^2 \theta (1 - \cos^2 \theta - 1)$$

$$\cos^2 \theta (\cos^2 \theta)$$

$$-\cos^4 \theta$$

$$15. \cos \theta + \sin \theta \tan \theta$$

$$\cos \theta + \frac{\sin \theta \cdot \frac{\sin \theta}{\cos \theta}}{1}$$

$$\frac{\cos \theta}{\cos \theta} + \frac{\sin^2 \theta}{\cos \theta}$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta}$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} =$$

$$\frac{1}{\cos \theta}$$

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⑤  $\sec \theta \cos \theta$       ⑦  $\cot^2 \theta - \csc^2 \theta$       ⑨  $\frac{\sin(-x)}{\cos(-x)}$   
 $\frac{1}{\cos \theta} \cancel{\cos \theta}$        $\cot^2 \theta - (\cot^2 \theta + 1)$        $\frac{-\sin(x)}{\cos(x)}$   
1       $\cot^2 \theta - \cot^2 \theta - 1$   
-1

⑥  $\tan \theta \cdot \csc \theta$       ⑧  $(1 - \cos^2 \theta) \csc \theta$       ⑩  $\frac{\sin(\pi/2 - x)}{\cos(\pi/2 - x)}$   
 $\frac{\sin \theta}{\cos \theta} \cancel{\frac{1}{\sin \theta}}$        $\sin^2 \theta \cdot \frac{1}{\sin \theta}$        $\frac{\cos(x)}{\sin(x)} = \cot(x)$   
1       $\cancel{\sin \theta}$

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(27)  $\cot\theta \sec\theta$

$$\frac{\cancel{\cos\theta}}{\sin\theta} \cdot \frac{1}{\cos\theta}$$
$$\frac{1}{\sin\theta}$$
$$\boxed{\csc\theta}$$

(29)  $\sin\theta(\csc\theta - \sin\theta)$

$$\sin\theta \csc\theta - \sin^2\theta$$

$$\cancel{\sin\theta} \cdot \frac{1}{\cancel{\sin\theta}} - \sin^2\theta$$

$$1 - \sin^2\theta$$
$$\boxed{\cos^2\theta}$$

(31)  $\frac{\cot\theta}{\csc\theta}$

$$\frac{\cancel{\cos\theta}}{\sin\theta}$$
$$\frac{1}{\sin\theta}$$

$$\frac{\cancel{\cos\theta} \cdot \cancel{\sin\theta}}{\sin\theta} \cdot \frac{1}{\cancel{\cos\theta}}$$
$$\boxed{\cos\theta}$$

(33)

$$\frac{1 - \sin^2\theta}{\csc^2\theta - 1}$$

$$\frac{\cos^2\theta}{\cot^2\theta}$$

$$\frac{\cos^2\theta}{\frac{\cos^2\theta}{\sin^2\theta}}$$
$$\sin^2\theta$$

$$\frac{\cos^2\theta \cdot \cancel{\sin^2\theta}}{\cancel{\cos^2\theta}}$$
$$\boxed{\sin^2\theta}$$

$$\begin{aligned} 5) \quad & \sec \theta \cdot \frac{\sin \theta}{\tan \theta} \\ & \frac{1}{\cos \theta} \cdot \sin \theta \cdot \cot \theta \\ & \frac{1}{\cos \theta} \cdot \sin \theta \cdot \frac{\cos \theta}{\sin \theta} \\ & \boxed{1} \end{aligned}$$

$$\begin{aligned} 37) \quad & \cos\left(\frac{\pi}{4} - \theta\right) \cdot \sec \theta \\ & \sin \theta \cdot \frac{1}{\cos \theta} \\ & \boxed{\tan \theta} \end{aligned}$$

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